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Integral assessment of phased investment scenarios of post-war development

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Abstract. The purpose of the article is to develop a theoretical and methodological model of the recovery of the national economy of Ukraine in the post-war period based on an integrated approach to the analysis of the investment process. The study used formalised economic and mathematical modeling, scenario analysis, elements of dynamic programming, and a critical review of current scientific literature on post-crisis management. The empirical basis was the analytical materials of international financial organisations on the assessment of damage and recovery needs. As a result, a three-phase structure of the transformation process (survival, reconstruction, growth) was formed, for each of which the corresponding accumulation integral $\int f(t) \cdot \omega(t)\, dt$ was built. The proposed model allows quantifying not only the volume of investments, but also the strategic feasibility of their placement over time. Three recovery scenarios were modelled: inertial, optimistic-coordinated and fragmented. The results showed that the synchronisation of investments with the phase logic of transformation, and not just their absolute volume, is a crucial factor in efficiency. The model also proved to be adaptable to changes in the pace of funding and allowed for the assessment of critical time windows for action. It made it possible to interpret investments not as one-off injections, but as a continuous process of strategic alignment. This approach opened up new analytical horizons for the development of sustainable economic recovery policies. The practical value of the model was the ability to identify critical time windows for the most effective investment, build phase budgets taking into account not only macro-financial volumes but also their temporal structure, adapt recovery policies to spatial and temporal asymmetries (regional phase analysis), and strengthen the role of international partners in shaping not just funding flows but their predictable temporal architecture

Keywords: post-war reconstruction; investment integral; scenario modelling; phase integral; investment strategy; time weighting function; strategic management

Introduction

The full-scale invasion of Ukraine by the Russian Federation in 2022 caused an unprecedented economic shock that transformed time from a background variable to a key factor in economic analysis. In the context of deep turbulence, time is not only a dimension of duration, but also a strategic resource that needs to be managed. The timeliness and coherence of investment flows in the postwar period determined not only the speed but also the quality of economic recovery. In this regard, there is a need to build an analytical model that would allow assessing the integral effect of investments, taking into account their time dynamics, and identifying optimal scenarios for the transition to growth.

According to the World Bank, the European Commission, and the UN, the direct costs of the war reached USD 135 billion, while the total needs for recovery and reconstruction were estimated at USD 411 billion

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(Poharska, 2023). Ukraine's economy suffered the deepest decline in the history of independence: real gross domestic product in 2022 decreased by 29.1%. This was accompanied by the destruction of critical infrastructure, the breakdown of logistics chains, the decline of industrial production, and the forced migration of millions of citizens (Voloshchuk *et al.*, 2025). At the same time, according to O. Poharska (2023), the adaptability of economic actors to new conditions allowed us to talk about the potential for a rapid transition to the recovery phase in the presence of a systematic investment approach.

The problem of economic recovery after shocks has been the subject of active research in recent years. In particular, S.T. Rachev *et al.* (2011) considered integral mathematical models in the context of market instability, demonstrating that the effectiveness of economic policy increases significantly when using time-effect aggregation operators. Their approach to the analysis of financial processes in a turbulent regime aimed at identifying the hidden temporal logic of crisis phenomena, which has direct application in the Ukrainian context. D.M. Cutler & E.L. Glaeser (2022) studied the geography of urban economic recovery after systemic crises. The authors concluded that the key role in recovery is played not so much by the volume of investment as by its distribution in time and space. This confirmed the thesis that it is necessary to take into account the phase logic of investment when building models of national-level recovery. Considerable attention was paid to the time dimension of economic transformations in the study by D. Acemoglu & P. Restrepo (2022), who modeled the long-term impact of automation on the US labor market. They used an integral approach to assess the effects that do not appear instantly but accumulate over time. This method allowed them to identify the delayed effects of structural changes – similar to the effects of investments in the Ukrainian economy in the post-crisis recovery phase.

Ukrainian scientist T.T. Kovalchuk (2020) proposed a model of economic and mathematical modeling under risk, which included the concept of "integral adaptation" the cumulative ability of the system to recover, which depends on the pace and structure of investment flows. This approach has also been used in simulations for post-conflict countries in Africa, which makes it relevant for Ukraine as well. According to P. Lecca *et al.* (2010), dynamic models of capital accumulation in the macroeconomy allowed not only to analyse current processes but also to predict development trajectories under different investment scenarios. Their study focused on the role of integrals in the formation of sustainable economies, which can be adapted to the phase recovery model. Researchers C. Ruza *et al.* (2019) studied the effects of interest rate burden on the fiscal sustainability of banking systems in developing countries. Using weighted integrals, they identified critical time frames for interventions in the financial system, demonstrating the value of time-dependent analysis in a crisis.

Also important was the study by V. Krey *et al.* (2014), which modeled the transformation of energy systems by 2100. Their model included a time weighting function that allowed for the identification of critical periods for investment, a concept that directly resonates with the concept of "efficiency windows" developed in this article. The purpose of this study was to create a conceptual model that interpreted the process of postwar recovery of Ukraine through the prism of the phase integral of investment. The use of the functional formula $\int f(t) \cdot \omega(t)dt$ allowed not only to formalise investment flows but also to take into account their temporal weight, i.e., the strategic importance of certain moments of time for each of the phases: survival, reconstruction, and growth.

Materials and Methods

This study used a comprehensive interdisciplinary approach that combined formal economic and mathematical modelling, scenario analysis, elements of dynamic programming, and a critical review of relevant scientific literature. This approach allowed to substantiate the conceptual and formal structure of the phase integrated model of investment in the context of post-crisis recovery. The theoretical basis was the concept of an integral as an aggregating tool in the context of temporal unevenness of impact. The economic-mathematical modelling was applied to construct a certain integral $\int f(t) \cdot \omega(t)\, dt$ as the basis of the analytical function of investment accumulation in each of the phases: survival, reconstruction, and growth. The modelling assumed that the function $f(t)$ described the intensity of the investment flow, while the weighting function $\omega(t)$ reflected the time priority or strategic importance of moment t for achieving the desired effect. This structure allowed us to move away from the traditional approach of analysing momentary values in favour of taking into account the dynamics of the accumulation of effects over time (Rachev *et al.*, 2011).

Scenario analysis was used to model three alternative recovery trajectories: inertial, optimistic-coordinated, and fragmented. To develop each scenario, the conditional parameters of the functions $f(t)$ and $\omega(t)$ were first set based on empirical assumptions about the pace, structure, and timing of investment inflows. In the fragmented scenario, the function $f(t)$ was modelled as a discrete wave with asynchronous pulses, and $\omega(t)$ was modelled as flat or non-systematic. For the optimistic scenario, a phase maximum in the function t was set in the reconstruction phase, which corresponded to the idea of a "window of opportunity" (Brundiers & Eakin, 2018).

For each scenario, we calculated conditional phase integrals: survival integral (S), recovery integral (R), and growth integral (G). These integrals allowed us to estimate how much investment was accumulated in the respective phase and to identify the dependence of efficiency on the synchrony of the functions $f(t)$ and $\omega(t)$. To calculate the integrals, we used numerical approximation methods (trapezoidal rule) implemented in Python with the numpy and scipy libraries. Dynamic programming elements were used to build a conditional matrix of transitions between phases. This made it possible to model at what volumes of the integral S it was possible to move to phase R without systemic losses, as well as to identify critical accumulation thresholds below which the launch of the next phase would be inefficient or risky.

To develop the model, a critical review of the current scientific literature on the topics of recovery economics, intertemporal planning, and phase investment was conducted. In particular, the approaches to macroeconomic modelling of economies in crisis were analysed, as outlined in D.M. Cutler & E.L. Glaeser (2022), D. Acemoglu & P. Restrepo (2022), V. Krey *et al.* (2014). The theoretical justification for intertemporal investment and the use of integrals in planning was provided by the concepts of F.P. Ramsey (1928), S.T. Rachev *et al.* (2011), T.T. Kovalchuk (2020). The ideas of sustainable recovery, behavioural consumption, and intertemporal choice were taken into account based on the works of K. Brundiers & H.C. Eakin (2018), O.P. Attanasio & G. Weber (1995). On their basis, the expediency of an integral approach to the analysis of economic dynamics in disasters was substantiated. As for the input materials, the generalised estimates of the international financial organisations World Bank (2023) and UNDP (n.d.), as well as analytical materials on Ukraine's recovery plans, in particular the report of the Recovery and Reconstruction Planning Platform (G7..., 2024), were used as an empirical basis. All data were used in the form of conditional approximations of investment flows broken down into phase intervals.

Results and Discussion

Integration as a tool of economic analysis had both formal and conceptual nature. The idea of treating economic changes as a set of small, continuous increments was developed in the classical intertemporal model of F.P. Ramsey (1928), which was later rethought through the prism of behavioural economics and dynamic programming. In modern scientific literature, integral approaches have been actively used in the context of analysing long-term investment processes. In S.T. Rachev *et al.* (2011), it was proposed to use integral operators to model market changes with regard to time disturbances. The authors proved that cumulative effects make sense only when the time factor is integrated into the formal structure of the model. The study by D.M. Cutler & E.L. Glaeser (2022) analysed the role of time lags in urban economic recovery. The authors emphasised the need to take into account inertial processes in investment policy and stressed that the formal integration of such delays can improve the accuracy of growth potential assessment. The defined integral was presented as a way to model the accumulated impact of investment flows over time.

The paper by D. Acemoglu & P. Restrepo (2022) use integral functions to describe long-term changes in employment as a result of the cumulative impact of technological transformation. This approach is an example of the use of variable-density integrals to measure the impact of innovation on macro parameters. In the Ukrainian context, the study by Kovalchuk (2020) is promising, analysing integral models for optimising investment decisions for economies recovering from disasters. The author introduces the concept of "integral adaptation" – a function that determines the accumulation of institutional resilience as a derivative of the volume and rhythm of investments in restored infrastructure. Thus, economic thought is increasingly turning to integral models as a way to capture the complex dynamic relationships between time, capital and effect. This approach is especially relevant in cases of structural disruption of the economic process, such as war, pandemic, or systemic recession. At the level of recovery theory, the key study was by K. Brundiers & H.C. Eakin (2018), which first formalised the concept of "windows of opportunity" in post-disaster periods. In this model, the integral efficiency of investment is maximised in a narrow time interval, which is consistent with the idea of the weighting function $\omega(t)$ in this model. Similarly, in V. Krey *et al.* (2014) used time weights in the analysis of the climate transformation of energy systems - the authors emphasised that the moment of implementation has a greater weight than the total cost.

The integral is also widely used in the analysis of intertemporal preferences and welfare. For example, in the intertemporal theory of consumption, the discounted integral of the utility function is used to determine the trajectory of rational choice O.P. Attanasio & G. Weber (1995). This model allowed to study the trade-offs between current and future welfare in the process of making strategic economic decisions. Thus, the integral in economics was not only an analytical tool, but also a conceptual bridge between point impact and systemic effect. Its application allowed us to formalise the ideas of cumulation, latent impact, inertia and lag, which were particularly important in the context of structural transformations, such as war or environmental crisis.

In the classical integral calculus, all elements of the subintegral domain had an equal contribution to the result – the function was integrated over the interval $[t_0; t_1]$ with equal weight. However, in economic analysis, it was extremely important to take into account the uneven importance of different time points. For this purpose, the concept of the weight function $\omega(t)$ was introduced, which allowed to differentiate the impact of a certain time period on the integral result. The integral with the weight function had the following general form:

$$I = \int_{t_0}^{t_1} f(t) \cdot \omega(t)\, dt, \tag{1}$$

where $\omega(t) \geq 0$ was a weighting function that reflects the priority of moment t.

Weight integrals have been widely used in finance to account for time preferences or risks. For example, in D. Debortoli *et al.* (2017) showed that welfare functions in macro-financial models take the form of integrals with an exponential weighting function. This function focused on the part of the time axis closer to the present. This design allowed political and economic agents to balance current pressures with a strategic long-term goal. In the framework of intertemporal optimisation, in particular in the analysis of dynamic household equilibrium, the weight function $\omega(t)$ was interpreted as a discount factor – that is, a subjective assessment of the value of the future (Krusell & Smith, 1998). In modern DSGE models, welfare integrals often have the form

$$\int U(c(t)) \cdot e\hat{}(-\rho t)\, dt, \quad (2)$$

where is the rate of intertemporal preference.

In the development of this logic, J. Crespo Cuaresma *et al.* (2020) focus on weighted investment indices that take into account not only the amount of capital but also the time of its placement. In the context of post-crisis recovery, they propose to use a weighting function $\omega(t)$ that increases during the reconstruction phase and peaks at the moment of structural break - when each invested unit has the highest multiplier effect.

The approach outlined in K. Brundiers & H.C. Eakin (2018), where $\omega(t)$ is used in modelling windows of opportunity in post-disaster recovery. In this interpretation, the weighting function took the form of a temporary impulse – a narrow interval $t \in [t_1; t_2]$, in which each investment had a maximum impact, after which its effectiveness rapidly declined. This approach was extremely relevant for Ukraine as a country in the reconstruction phase. In post-war planning, these approaches will help determine exactly when to invest – not just how much. Post-crisis economies have functioned in disparate phases, each of which requires a different pace, structure and purpose of investment. To formalise these phases, it was necessary to introduce three integral aggregates of resource time: survival integral, recovery integral and growth integral. They allowed not only to quantify cumulative investment efforts in each period, but also to determine their systemic function.

Firstly, the Survival integral, denoted as $S = \int_{t0}\hat{}_{t1} f_s(t)\, dt$, accumulated resource flows aimed at supporting the functioning of critical systems: medical, humanitarian, and security. Its economic sense was to accumulate the minimum amount of resources necessary to prevent systemic collapse. In this context, it was appropriate to analyse the dynamic resilience model K. Martínez *et al.* (2025), where the system's survival is linked to the lower bound of the resource flow required to maintain basic functionality. Secondly, the Recovery integral, $R = \int_{t1}\hat{}_{t2} f_r(t)\, dt$, reflected the recovery and reorganisation phase, when resources were directed to the resuscitation of economic infrastructure, the financial system, and domestic production.

This stage was not a direct continuation of the previous one, but rather a transition to a new economic configuration. According to the approach of A. Bénassy-Quéré & B. Weder di Mauro (2020), the recovery had to be not only cumulative but also "qualitatively driven" – that is, it had to be aimed at structural transformation. Third, the Growth integral, $G = \int_{t2}\hat{}_{t3} fg(t)\, dt$, aggregated the investment flows that triggered long-term economic growth. He envisaged a change in the logic of investment: from emergency to systemic, from reactive to strategic. In G. Schwartz *et al.* (2020) emphasised that effective growth after a shock is only possible if investment is targeted to take into account climate, institutional, and social parameters – that is, the growth integral has become a function of many vectors. All three integrals could be presented as phase components of the overall development integral:

$$I\Sigma = S + R + G = \int_{t0}\hat{}_{t1} f_s(t)\, dt + \int_{t1}\hat{}_{t2} f_r(t)\, dt + \int_{t2}\hat{}_{t3} fg(t)\, dt. \quad (3)$$

The formalisation of this approach allowed not only to describe the trajectory of post-crisis recovery, but also to model the policy of resource allocation over time in accordance with strategic priorities. It was important to note that the phase division was not purely chronological, but also structural: each phase required separate institutional mechanisms for implementation and performance evaluation. Thus, the introduction of the three phase integrals allowed us to move from a linear view of recovery to a multi-level, integral architecture of economic time. This opened up the possibility of adaptive resource management in the face of radical uncertainty. The next logical step was to build the formal structure of an integral model of investment reconstruction in Ukraine, which mathematically and conceptually combined what had been introduced earlier (survival, recovery, growth integrals) into a single economic and mathematical framework. The model was based on the notion of post-war economic recovery as a sequence of phases with different structures of investment flows accumulated over time. The general integral expression was as follows:

$$I\Sigma = \int_{t0}\hat{}_{t1} f_s(t) \cdot \omega_s(t)\, dt + \int_{t1}\hat{}_{t2} f_r(t) \cdot \omega_r(t)\, dt + + \int_{t2}\hat{}_{t3} fg(t) \cdot \omega g(t)\, dt, \quad (4)$$

where $f_s(t)$, $f_r(t)$, $fg(t)$ are functions of investment flows in the phases of survival (S), recovery (R) and growth (G), respectively; $\omega_s(t)$, $\omega_r(t)$, $g(t)$ are weight functions that reflect the priority of time points in each phase; $[t_0; t_3]$ is the time horizon of post-war reconstruction.

The key in the model was not only the amount of resources invested, but also their distribution over time, as the impact of f(t) depended on the weighting function $\omega(t)$, which is determined by the specifics of needs and windows of efficiency. This was in line with the idea of time-sensitive accumulation, first substantiated in the study by M. Forni & L. Gambetti (2016), where the authors found that the distribution of stimuli over time had a much greater impact on long-term GDP than the size of the fiscal package itself. In the first phase (survival), $\omega_s(t)$ had the form of a dampened function, which decreased the priority of spending over time, while $f_s(t)$ was usually an exogenous variable determined by humanitarian aid. In the second phase, $\omega_r(t)$ took the form of an impulse function with a maximum at time t^*, which corresponded to the "window of opportunity" for the restoration of critical infrastructure T. Ylä-Anttila *et al.* (2023). The third phase was described by $g(t)$, which was an increasing function, as the efficiency of each additional investment increased with the restoration of market mechanisms.

In contrast to the classical models of R.M. Solow (1956) or F.P. Ramsey (1928), the proposed construction assumed that the function *f*(*t*) was not smooth but had a fragmented structure due to the shock nature of war and political lags in financing. In this context, it was appropriate to use the Lebesgue integral, which allowed aggregating *f(t)* even in the presence of discrete jumps in the investment flow. This approach was in line with modern practices in modelling economic processes with discrete shocks, where traditional integration methods might not be sufficient for accurate analysis (Tao, 2010; Nunes & Pimentel, 2015). Taken together, the model allowed us to quantify the minimum funding threshold for the transition between phases, model the impact of delayed investment in the recovery phase on future growth (i.e., the derivative of G in t_2), and determine the optimal form of $\omega(t)$ for each phase by minimising welfare losses or delayed effects.

This approach allows political and economic actors (government, donors, development banks) to synchronise investment flows with the institutional dynamics of the system and the logic of strategic development. The high adaptability of the model also made it possible to incorporate additional weighting factors: risk ($\rho(t)$), social effect ($\sigma(t)$), climate impact ($\kappa(t)$), etc. The next step was scenario modelling – how different profiles f(t) and weighting functions $\omega(t)$ within the survival, recovery, and growth phases affected the result of the IΣ integral. A formalised scenario analysis based on economic logic was carried out. This made it possible to show how variations in *f(t)* and $\omega(t)$ affected the amount of accumulated investment in each phase and the overall recovery effect.

The integral reconstruction model has emerged not only as an analytical construct, but also as an applied tool for forecasting the results of investment strategies in the time perspective. To verify its applied potential, three conditional scenarios are considered: the inertial scenario (I): investment flows *f(t)* are unevenly distributed, with concentration in the later phases ($t_2 \rightarrow t_3$), weight functions $\omega(t)$ have low sensitivity to time; the optimal scenario (O): *f(t)* is balanced in accordance with the priority of phases, weight functions $\omega(t)$ have a pronounced maximum in the respective phase centres; fragmented scenario (F): investments are impulsive and random, *f(t)* is discrete, $\omega(t)$ does not correlate with the moments of structural needs. The calculation of the IΣ integral in each scenario (with conditional equations *f(t)* and $\omega(t)$ that can be adapted to the model simulator) showed the critical importance of the time factor. In the F scenario, the value of IΣ could even exceed the optimal scenario, but would have a smaller systemic effect due to the disturbed cumulation.

From an applied point of view, scenario modelling has made it possible to predict the costs of delays or lack of coordination: in real time, this is reflected in lost opportunities to rebuild infrastructure, loss of donor confidence, or irreversible decline in human capital potential. This model could also be the basis for building digital tools to support government planning, as was done in the Recovery Plan Simulator for the Balkan countries, OECD (2022). The integral model, based on phase functions *f(t)* with appropriate weighting coefficients $\omega(t)$, allowed for comparative scenario modelling of recovery. This approach has been widely used in studies by the World Bank, IMF, and UNDP for countries recovering from military or environmental shocks (World Bank, 2023; UNDP, n.d.). Therefore, it was appropriate to present a conditional simulation based on three scenarios:

The first was the inertial scenario for minimal and delayed investment. In this case, the functions $f_s(t)$, $f_r(t)$, *fg(t)* were low-amplitude, and $\omega(t)$ was either constant or growing late. The accumulated survival integral was sufficient to avoid systemic collapse, but the recovery integral was underestimated and stretched in time. Consequently, the growth integral was shifted to the future and has a low efficiency ratio. This scenario was described in the study by G. Ferrari (2016) on the post-conflict economy of South Sudan, where the delay in recovery led to the loss of the "window of transformation". The second is an optimistic scenario, namely phase-coordinated investment. Here, the function f(t) had a maximum in phase R, and $\omega(t)$ was constructed as an impulse function with a time focus in t_1-t_2. This corresponded to the "build back better" scenario of the post-war reconstruction strategy in Croatia or Bosnia, when large-scale and rapid investment in infrastructure had a cumulative effect on the entire economy. In the simulation, this scenario showed maximum R values and high G growth already on the medium-term horizon (Callegaro *et al.*, 2019).

The third was a fragmentary scenario for uncoordinated sources and asynchronous phases. This variant was characterised by a wave-like structure of *f(t)* and $\omega(t)$, with local extrema that do not coincide in time. For example, large donor injections in phase G in the absence of adequate S or R created systemic inefficiency. The S and R integrals did not provide a foundation, so even large late investments lose their multiplier effect. This approach was modelled in S. Barakat (2009), where it was shown that chaotic post-crisis investment in MENA countries led to "institutional decay". The volume of phase integrals of survival, recovery and growth under alternative development scenarios is presented in Table 1.

Table 1. Phase integrals, priority and expected effects as post-war recovery scenarios

Scenario	$\int f_s(t)\,dt$ (S)	$\int f_r(t)\,dt$ (R)	$\int fg(t)\,dt$ (G)	**Expected effect**
Inertial	~ sufficiently	~insufficiently	~ postponed	Slow growth
Optimistic	adequately	maximum	active	Structural reconstruction
Fragmentary	chaotically	failure	late/ineffective	No growth occurs

Source: compiled by the authors

The table shows that the volume of integrals was not decisive in itself: the critical factor was the relationship between the moment of maximum *ω(t)* and the efficiency *f(t)* at that moment. That is, G depends on the synchrony of the phases, not just the sum of the inputs. Three graphs (Fig. 1) showed the phase structure of investment flows *f(t)* in three scenarios: inertial – slow, uncoordinated financing with a delayed effect, optimistic – coordinated and timely investment in all phases, fragmented – chaotic structure with asynchrony between phases.

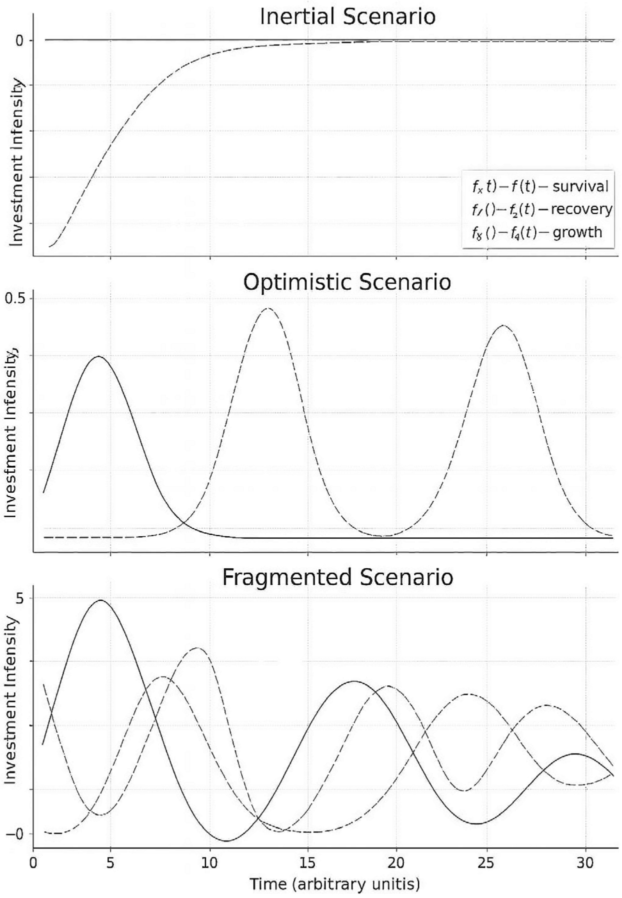

Figure 1. Graphs of three scenarios *f*(*t*) with superimposed *ω*(*t*)

Source: compiled by the authors

The next step was the political and strategic interpretation of the model, which allowed us to translate the formalised integral construct (survival, recovery, growth integrals) into the policy plane, namely to answer the question of how this model could be used to develop real strategies at the level of the state, international partners and regions. The integral model of investment reconstruction allowed not only to quantitatively model the post-crisis transition, but also to formulate strategic recommendations for the allocation of resources over time. Its key value for public policy lies in its ability to make visible the critical relationship between the timing of investment, institutional capacity, and economic inertia. At the level of public policy, the model required the creation of a mechanism for phased recovery management. Each phase (survival, recovery, and growth) required not only different amounts of funding but also specific institutions. Survival integral was implemented through rapid response mechanisms: National Platform for Humanitarian Coordination, support for internally displaced persons, and reserve funds. The Recovery integral was to be structured through the creation of a reconstruction investment framework focused on the effective use of external sources. This was in line with the UNOPS (2025) approach, which recommended the creation of Recovery Coordination Units as separate governing bodies under the Cabinet of Ministers.

In the growth phase, according to the recommendations of the G7 Multi-agency Donor Coordination Platform for Ukraine (2024), it was critical to institutionalise Growth integral in the form of medium- and long-term structural transformation plans: tax system reform, digitalisation of registers, and transparency of concessions. All three phases had to be stitched together into a single time programme with the logic of continuity and priority *ω(t)*. The integral model directly pointed to the critical role of rhythmicity and predictability of external financing. In the study by B. Eichengreen & V. Rashkovan (2025) emphasised that most reconstruction failures in post-conflict countries were caused not by a lack of funds, but by the asynchrony of their flow. Approaches based on a "lagging infusion" without taking into account the time weighting function proved to be ineffective.

Therefore, international partners had to move from an ex post grant mechanism to an ex ante budget programming system with clear time mandates. The *ω(t)* model had to be aligned with macrofinancial stabilisation policies, in particular, under the IMF's Extended Fund Facility 2023-2027 programme. As noted in the European Investment Bank's report (2025), time structured investment yielded higher returns than volume. The application of integral logic at the regional level made it possible to identify areas with a critical gap or excessive concentration of resources in one phase. This opened up the possibility of creating a phase recovery index – the S:R:G ratio in each region, with further optimisation of distribution. In the work of S. Hanandeh *et al.* (2018) proposed the use of spatial-weight integrals as a way to prioritise infrastructure restoration in a resource-limited environment. Regional administrations could use such models to submit funding requests that are aligned with the national phase logic. This created transparency and competition for investment resources based on analytical indicators rather than administrative pressure. A diagram that visualises the strategic synchronisation of the investment reconstruction phases according to the integral model, namely the survival phase (S) – the first 10 units of time, the recovery phase (R) – the next 10, and the growth phase (G) – the final 10, is presented in Table 2.

Table 2. Phase synchronisation of investment reconstruction

Recovery phase	Actors	Tools	Type of control
Survival Integral *(humanitarian needs)*	- International organisations - Government services - Volunteer networks	- Direct humanitarian aid - Emergency budgets	Reactive management
Recovery Integral (system rehabilitation)	- Ministries - Recovery agencies - State-owned banks	- Rebuilding infrastructure - Programme lending - Energy modernisation programmes	Strategic planning
Growth Integral (structural growth)	- Private sector - Local investors - Development clusters	- PPP - Capital investments - Innovative projects	Visionary development management

Source: compiled by the authors

Thus, the results of the modelling demonstrated the high sensitivity of the integrated phase indicators to the synchronisation of investment flows with time priorities. Regardless of the volume of *f(t)*, it was the shape and shift of the weighting function *ω(t)* that determined in which phase the main recovery effect would be concentrated. This led to the conclusion that effective management of post-crisis investments is not only about securing funding, but also about its correct rhythmic allocation over time. These results have laid the groundwork for the development of new strategic approaches to planning the recovery economy, taking into account its phased nature.

Conclusions

The integral model of investment reconstruction developed in this article offers a fundamentally new framework for understanding the post-war economic development of Ukraine. Its main intellectual innovation is the transition from the analysis of momentary (static) parameters to the model of accumulation of the effect over time. In the context of the catastrophe that destroyed the linearity of economic dynamics, it is the integral – as a mathematical and conceptual tool – that allows us to record not only the volume of resources, but also their strategic synchronisation. In the framework of the study, an integral model of phase investment was built, which allows assessing the effectiveness of economic recovery, taking into account the time dynamics. A three-phase structure of recovery was formulated: survival, reconstruction and growth, each of which was characterised by a separate time interval and specific dynamics of investment needs. For each phase, a separate integral was constructed in the form *∫f(t)·ω(t) dt*, where the function *f(t)* described the intensity of the investment flow, and *ω(t)* was the time weight reflecting the priority of the moment of implementation. Thus, the proposed approach provided a quantitative assessment of not only the volume, but also the feasibility of investments, taking into account their placement in time.

Using scenario analysis, three conditional scenarios were modelled: inertial, fragmentary and optimistic (coordinated). For each scenario, conditional phase integrals were calculated, which made it possible to assess the extent to which different structure *ω(t)* affects the recovery efficiency. It was confirmed that even with the same *f(t)*, the efficiency varied significantly depending on the synchrony with *ω(t)*, which proved the crucial role of rhythmic and timely investment. The study also confirmed the suitability of the Lebesgue integral for aggregating discrete investment spikes, which was particularly relevant in the context of post-war financing, which was characterised by uneven revenues. This made it possible to provide adequate mathematical treatment of even fragmented flows characteristic of war and crisis delays. Further research involves the empirical formalisation of *ω(t)* and the development of an optimisation unit for finding investment trajectories in a changing environment.

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None.

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Інтегральна оцінка фазових інвестиційних сценаріїв післявоєнного розвитку

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Анотація. Метою статті була розробка теоретико-методологічної моделі відновлення національної економіки України в поствоєнний період на основі інтегрального підходу до аналізу інвестиційного процесу. У дослідженні було використано формалізоване економіко-математичне моделювання, сценарний аналіз, елементи динамічного програмування, а також критичний огляд актуальної наукової літератури з питань післякризового управління. Емпіричною основою слугували аналітичні матеріали міжнародних фінансових організацій щодо оцінки збитків і потреб у відновленні. У результаті було сформовано трифазну структуру процесу трансформації (виживання, реконструкція, зростання), для кожної з яких побудовано відповідний інтеграл накопичення $\int f(t) \cdot \omega(t)\, dt$ Запропонована модель дозволила кількісно оцінити не лише обсяг інвестицій, але й стратегічну доцільність їхнього розміщення в часі. Було проведено моделювання трьох сценаріїв відновлення: інерційного, оптимістично-скоординованого та фрагментарного. Результати показали, що вирішальним фактором ефективності є синхронізація інвестицій з фазовою логікою трансформації, а не лише їх абсолютний обсяг. Модель також виявилася адаптивною до змін у темпі фінансування та дозволила оцінювати критичні часові вікна для дій. Вона забезпечила можливість інтерпретації інвестицій не як одноразових вливань, а як безперервного процесу стратегічного вирівнювання. Такий підхід відкрив нові аналітичні горизонти для розробки політик сталого економічного відновлення. Практична цінність моделі полягає в можливості визначення критичних часових вікон для максимально ефективного інвестування, побудови фазових бюджетів з урахуванням не тільки макрофінансових обсягів, а й їх темпоральної структури, адаптації політики відновлення до просторової й часової асиметрії (регіональний фазовий аналіз), посилення ролі міжнародних партнерів у формуванні не просто потоків фінансування, а їхньої передбачуваної часової архітектури

Ключові слова: післявоєнне відновлення; інтеграл інвестицій; сценарне моделювання; фазовий інтеграл; інвестиційна стратегія; вагова функція часу; стратегічне управління